AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

AS
FURTHER MATHEMATICS

Paper 2 Mechanics

Thursday 16 May 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Statistics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A turntable rotates at a constant speed of $33\frac{1}{3}$ revolutions per minute.

Find the angular speed in radians per second.

Circle your answer.

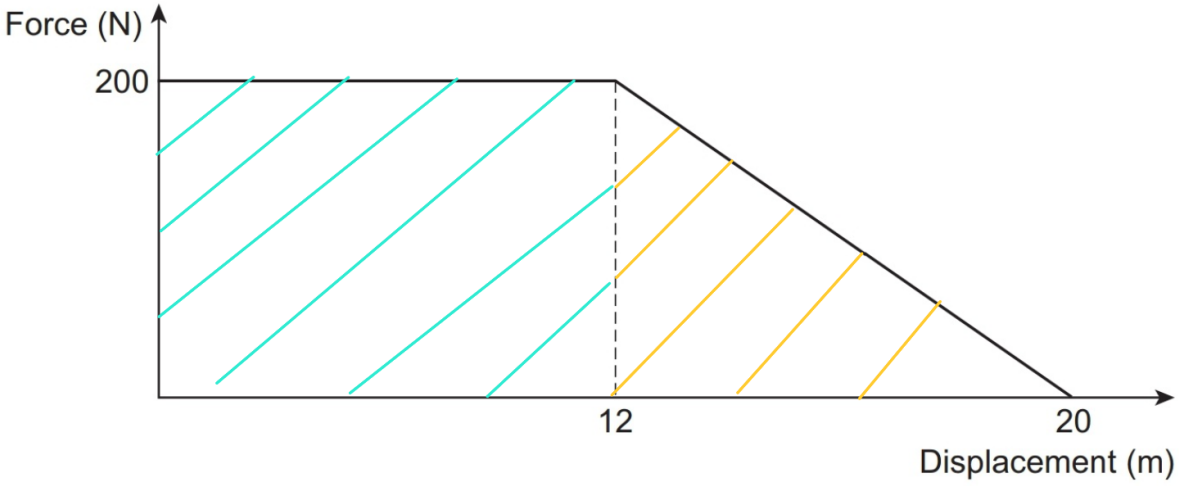
[1 mark]

$\frac{5\pi}{9}$ $\quad$ (**$\frac{10\pi}{9}$**) $\quad$ $\frac{5\pi}{3}$ $\quad$ $\frac{20\pi}{9}$

$33\frac{1}{3}$ rpm $= \frac{33\frac{1}{3}}{60} = \frac{5}{9}$ revs per second

as one rev $= 2\pi$ rads, rads per second $= 2\pi \times \frac{5}{9} = \frac{10\pi}{9}$

2 The graph shows the resistance force experienced by a cyclist over the first 20 metres of a bicycle ride.



Find the work done by the resistance force over the 20 metres of the bicycle ride.

Circle your answer.

[1 mark]

1600 J $\quad$ 3000 J $\quad$ (**3200 J**) $\quad$ 4000 J

Work done = area under graph

$= 200 \times 12 + \frac{200 \times (20 - 12)}{2}$

$= 3200$ J

3 A formula for the elastic potential energy, E, stored in a stretched spring is given by

$$E = \frac{kx^2}{2}$$

where x is the extension of the spring and k is a constant.

Use dimensional analysis to find the dimensions of k.

[3 marks]

$[E] = M L^2 T^{-2}$

and $E = \frac{k(L)^2}{2}$:

$[k](L)^2 = M L^2 T^{-2}$

$[k] = M T^{-2}$

4 **In this question use $g = 9.8\,\text{m s}^{-2}$**

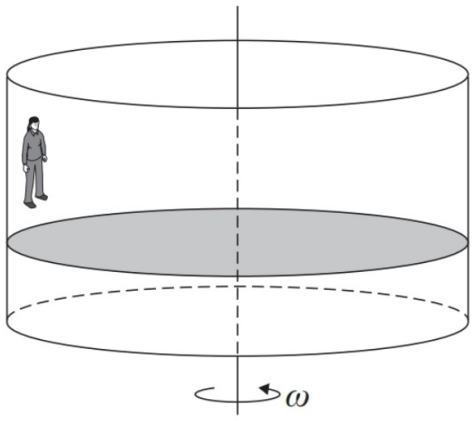
A ride in a fairground consists of a hollow vertical cylinder of radius 4.6 metres with a horizontal floor.

Stephi, who has mass 50 kilograms, stands inside the cylinder with her back against the curved surface.

The cylinder begins to rotate about a vertical axis through the centre of the cylinder.

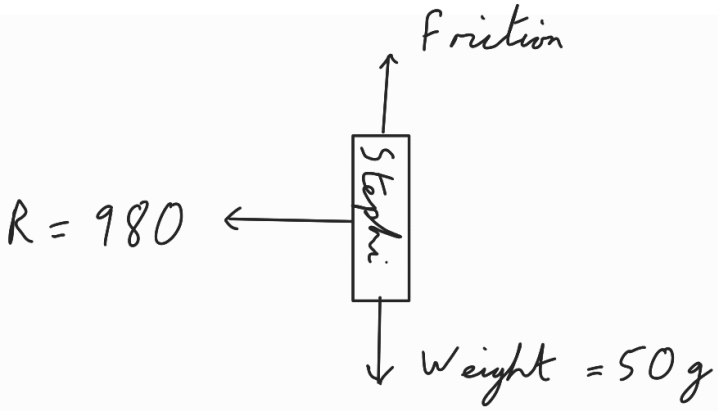
When the cylinder is rotating at a constant angular speed of ω radians per second, the magnitude of the normal reaction between Stephi and the curved surface is 980 newtons.

The floor is lowered and Stephi remains against the curved surface with her feet above the floor, as shown in the diagram.



4 (a) Explain, with the aid of a force diagram, why the magnitude of the frictional force acting on Stephi is 490 newtons.

[2 marks]



To stop her sliding down the wall, the upward frictional force must equal the downward weight force.

∴ Friction = 50g = 490 newtons.

4 (b) Find ω

[3 marks]

Force towards circle centre $= mr\omega^2$

$$980 = 50(4.6)\omega^2$$

$$\omega = \sqrt{\frac{980}{50 \times 4.6}}$$

$$\omega = 2.064\ldots$$

$$= 2.1 \text{ rad } s^{-1}$$

4 (c) State one modelling assumption that you have used in this question.

Explain the effect of this assumption.

[2 marks]

It is assumed that air resistance is negligable.
This allows the assumption that all friction is vertical.

or

Radius is assumed to be exactly 4.6 m everywhere, meaning there is no need to consider variations in the value of the radius

5 A car of mass 1000 kg has a maximum speed of 40 m s^{-1} when travelling on a straight horizontal race track.

The maximum power output of the car's engine is 48 kW

The total resistance force experienced by the car can be modelled as being proportional to the car's speed.

Find the maximum possible acceleration of the car when it is travelling at 25 m s^{-1} on the straight horizontal race track.

Fully justify your answer.

[7 marks]

Power $= fv$

$R \propto v \Rightarrow R = kv$

$\therefore$ At 40 ms^{-1} : $P = 48000 = 40f$

$R = 40k$

At max speed, $\Sigma f = 0$, $\therefore R = f$

$f = \frac{48000}{40} = 1200\,N$

$k = \frac{R}{40} = \frac{1200}{40} = 30$

At 25 ms^{-1} :

$R = 30 \times 25 = 750$

$P = 48000 = 25D$ ← driving force

$D = 1920\,N$

By Newton II :

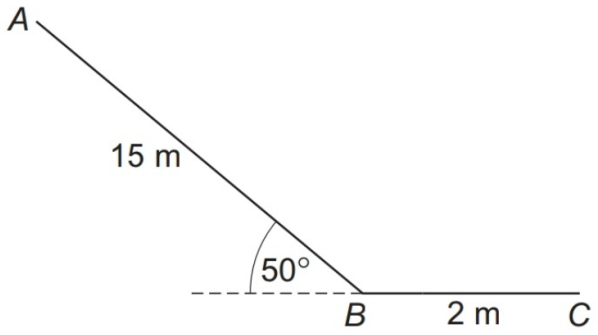
$1920 - 750 = 1000a$

$a = 1.2\ ms^{-2}$

6 **In this question use $g = 9.8\,m\,s^{-2}$**

Martin, who is of mass 40 kg, is using a slide.

The slide is made of two straight sections *AB* and *BC*.
The section *AB* has length 15 metres and is at an angle of 50° to the horizontal.
The section *BC* has length 2 metres and is horizontal.



Martin pushes himself from *A* down the slide with initial speed $1\,m\,s^{-1}$
He reaches *B* with speed $5\,m\,s^{-1}$

Model Martin as a particle.

6 (a) Find the energy lost as Martin slides from *A* to *B*.

[4 marks]

PE at A: $mgh = 40g(15\sin 50) = 4504$

KE at A: $\frac{1}{2}mv^2 = \frac{1}{2} \times 40 \times 1^2 = 20$

KE at B: $\frac{1}{2}mv^2 = \frac{1}{2} \times 40 \times 5^2 = 500$

$\therefore$ Loss of energy $= 4504 + 20 - 500 = 4024$

$= 4000\ J$ (2 sf)

6 (b) Assume that a resistance force of constant magnitude acts on Martin while he is moving on the slide.

6 (b) (i) Show that the magnitude of this resistance force is approximately 270 N

[2 marks]

ΔE = force × distance

$4024 = 15f$

$\therefore f = 268.2\ldots$ N

$f = 270$ N (2sf) (as required)

6 (b) (ii) Determine if Martin reaches the point *C*.

[3 marks]

KE at B = force × distance travelled

$500 = 270d$

$d = 1.85$

$1.85 < 2$

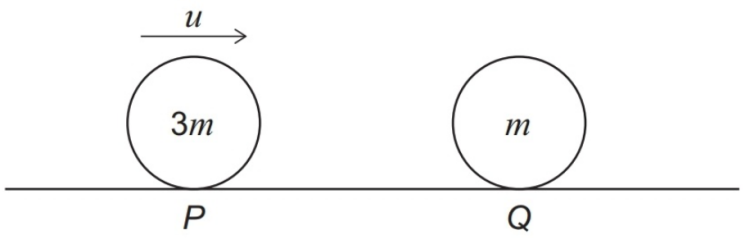
If Martin's size negligible, he will not reach the end.
If his size is not negligible, he may make it.

7 Two smooth spheres, P and Q, of equal radius are free to move on a smooth horizontal surface.

The masses of P and Q are $3m$ and m respectively.

P is set in motion with speed u directly towards Q, which is initially at rest.

P subsequently collides with Q.



Immediately after the collision, P moves with speed v and Q moves with speed w.

The coefficient of restitution between the spheres is e.

7 (a) (i) Show that

$$v = \frac{u(3-e)}{4}$$

[4 marks]

Conservation of momentum:

$$3mu + 0 = 3mv + mw$$
$$3u = 3v + w \quad -(1)$$

Newton's law of restitution:

$$e = \frac{w-v}{u-0}$$

$$w - v = ue \quad -(2)$$

(1) − (2): $3u - ue = 3v + w - (w - v)$

$$u(3-e) = 4v$$

$$\therefore v = \frac{u(3-e)}{4} \quad \text{(as required)}$$

7 (a) (ii) Find w, in terms of e and u, simplifying your answer.

[2 marks]

$w - v = ue$

$w - \frac{u(3-e)}{4} = ue$

$w = \frac{3u - ue}{4} + \frac{4ue}{4}$

$w = \frac{3u(1+e)}{4}$

7 (b) Deduce that

$$\frac{u}{2} \leq v \leq \frac{3u}{4}$$

[2 marks]

$0 \leq e \leq 1$

at $e = 0$:

$v = \frac{u(3-0)}{4} = \frac{3u}{4}$

at $e = 1$:

$v = \frac{u(3-1)}{4} = \frac{u}{2}$

hence: $\frac{u}{2} \leq v \leq \frac{3u}{4}$

7 (c) (i) Find, in terms of m and u, the maximum magnitude of the impulse that P exerts on Q.

[3 marks]

Impulse $= \Delta m v$

so no energy goes into deformation.

maximum occurs at $e = 1 \Rightarrow v = \frac{u}{2}$

$\therefore I = 3m\left(\frac{u}{2}\right) - 3mu$

$I = -\frac{3mu}{2}$

Magnitude $= \frac{3mu}{2}$

7 (c) (ii) Describe the impulse that Q exerts on P.

[1 mark]

Impulse from Q on P is equal in magnitude but opposite in direction.